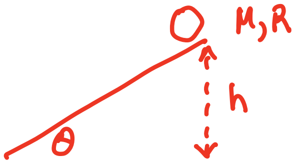
NAME: KEY

Rotation Problems III

1. Derive an expression for the speed of a solid uniform sphere of mass M and radius R after rolling down a hill. The sphere starts from a height h, and the hill has a base angle θ. Does your answer depend on the size of the sphere or the angle of the hill?



$\Sigma E_i = \Sigma E_f$

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ & $v = R\omega$

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left[\frac{2}{5}MR^2\right]\omega^2$

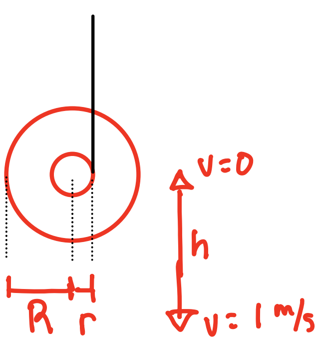
$Mgh = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2$ $(v^2 = R^2\omega^2)$

$gh = \frac{7}{10}v^2$

$\boxed{v = \sqrt{\frac{10}{7}gh}}$

θ, R & M don't matter! Just h and the shape.

2. A yo-yo can be thought of as being two uniform disks, each of radius 5 cm and mass 150 grams. The string is would around a small post of negligible mass and moment of inertia, but radius 1 cm. Starting from rest, the yo-yo falls a distance h and reaches a final speed of 1 m/s. What was h?



$m = 0.15$ kg
$r = 0.01$ m
$R = 0.05$ m

$\Sigma E_i = \Sigma E_f$

$2(mgh) = \cancel{2}\left(\frac{1}{\cancel{2}}mv^2\right) + \cancel{2}\left(\frac{1}{\cancel{2}}I\omega^2\right)$ & $v = r\omega$

the little radius!

$2mgh = mv^2 + \left[\frac{1}{2}mR^2\right]\omega^2$

big radius

$2mgh = mv^2 + \frac{1}{2}mR^2\left(\frac{v^2}{r^2}\right)$ $\omega^2 = \frac{v^2}{r^2}$

$2gh = v^2 + \frac{R^2}{2r^2}v^2$

$h = \frac{v^2}{2g}\left(1 + \frac{R^2}{2r^2}\right)$

$h = \frac{(1)^2}{2(10)}\left(1 + \frac{(.05)^2}{2(.01)^2}\right)$

$\boxed{h = 0.675 \text{ m}}$

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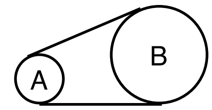
3. Two wheels are connected by a belt that does not slip. The radius of one wheel (B) is three times the radius of the second (A). What would be the ratio of the rotational inertias I_A/I_B if



a. both wheels had the same angular momentum?

$L_A = L_B$ & $v_A = v_B$ (because belt has 1 speed)

$I_A \omega_A = I_B \omega_B$ So $r_A \omega_A = r_B \omega_B$

$I_A (3\omega_B) = I_B \omega_B$ $\omega_A = \frac{r_B}{r_A}\omega_B$ & $r_B = 3r_A$

So $\boxed{\frac{I_A}{I_B} = \frac{1}{3}}$ $\therefore \underline{\omega_A = 3\omega_B}$

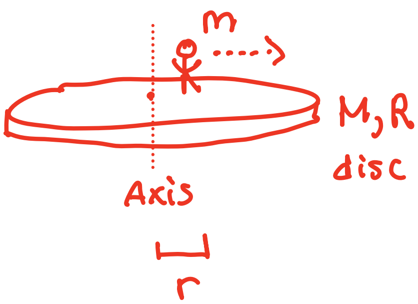
b. both wheels had the same rotational kinetic energy?

$K_A = K_B$

$\frac{1}{2} I_A \omega_A^2 = \frac{1}{2} I_B \omega_B^2$

so $I_A (3\omega_B)^2 = I_B \omega_B^2 \rightarrow \boxed{\frac{I_A}{I_B} = \frac{1}{9}}$

4. A child of mass 35 kg is sitting on a large rotating disk (100 kg and radius 2 m) in a playground. The disk is rotating at 1 revolution per second, and the child is initially sitting 0.5 meters from the center. The child carefully crawls to the edge of the disk. What is the new rotation rate?



$L_i = L_f$ $(\Sigma\tau = 0)$

$[I_{disc} + I_{child}]\omega_i = [I_{disc} + I_{child}]\omega_f$

$[\frac{1}{2}MR^2 + mr^2]\omega_i = [\frac{1}{2}MR^2 + mR^2]\omega_f$

starts @ r ends @ R

$$\omega_f = \frac{[\frac{1}{2}MR^2 + mr^2]}{[\frac{1}{2}MR^2 + mR^2]}\omega_i = \frac{[\frac{1}{2}(100)(2)^2 + (35)(.5)^2]}{[\frac{1}{2}(100)(2)^2 + (35)(2)^2]}(1)$$

$\boxed{\omega_f = 0.614 \text{ rps}}$

Rotation Problems III

5. You are sitting on a rotating stool with your arms held out. If you pull your arms in, your rotational speed increases.
   a. Are there any external forces exerted on you during this process? If so, list them.

   Yes! Normal Force and weight

   b. Are there any external torques exerted on you during this process? If so, list them.

   No! Both of those forces are ∥ to rotation Axis.

   c. What happens to your moment of inertia in this process? Explain.

   I will DECREASE because you are pulling mass closer to rotational axis.

   d. Why does your rotational speed increase?

   Because L is conserved (b/c $\Sigma\tau = 0$)

   So $I_{initial}\,\omega_{initial} = I_{final}\,\omega_{final}$

   e. What happens to your kinetic energy in this process? Explain.

   It actually goes up! Since $I_i\omega_i = I_f\omega_f$, $I_i\omega_i^2 \neq I_f\omega_f^2$

   Because K depends on velocity squared, the higher velocity will have higher K. [The energy comes from the chemical potential energy in your body. In a sense, you had to do positive work on your hands to pull them in, so you gave your hands some kinetic energy.]

Answers:

1. $\boxed{}$ *; does not depend on angle or size of sphere*

2. h = 0.68 m *3. a.* $I_A{:}I_B = 1{:}3$ *b.* $I_A{:}I_B = 1{:}9$ *4. 0.61 rps*

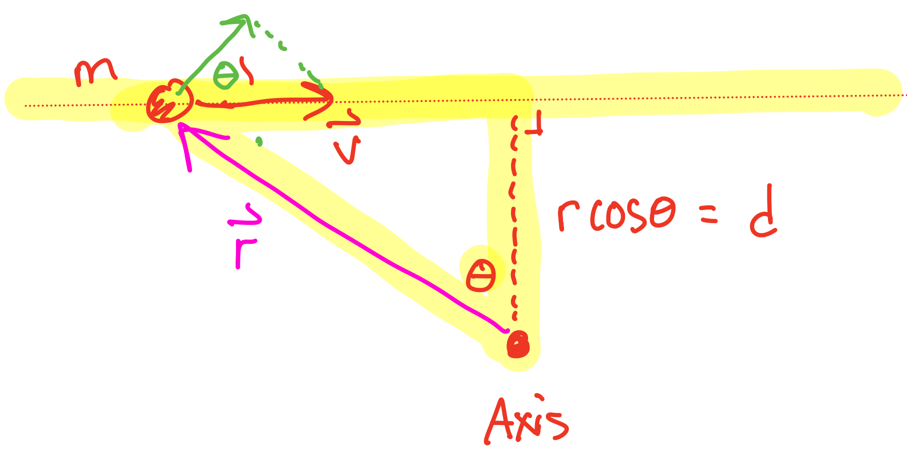
Angular Momentum

$p = mv \longrightarrow L = I\omega$

mostly use this

units

$\frac{kg \cdot m^2 \cdot rad}{s}$



$\vec{l} = \vec{r} \times \vec{p}$

$l = r p_\perp$

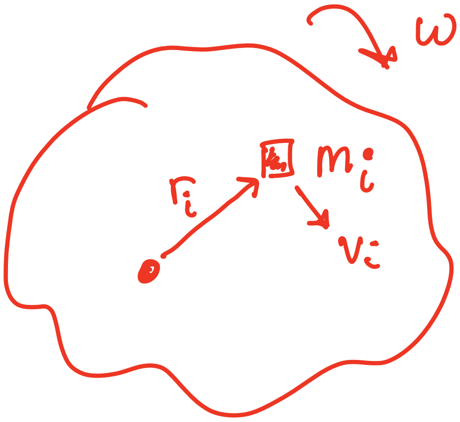
$l = r m v_\perp$

$l = r m v \cos\theta$

$l = mvd$ hey! that's constant!

$\vec{l} = \vec{r} \times \vec{p}$

real definition



$$v_i = r_i \omega$$

$$\vec{L} = \sum_i \vec{l}_i$$

$$L = \sum_i r_i p_i$$

$$= \sum r_i m_i v_i$$

$$= \sum r_i m_i (r_i \omega)$$

$$= \left(\sum m_i r_i^2 \right) \omega$$

hey! That's I!

$$L = I \omega$$